INTERNATIONAL DAY OF MATHEMATICS
MARCH 14

Melting the Ice and Orders of Magnitude

Participants:

Ages 12 and up.

Overview:

This IDM activity has two independent parts. Both deal with big questions about things that can happen to our planet. In the first activity questions are focused on the problem of sea level rise due to global warming. In the second activity the discussion is more general about how estimation problems related to our planet can be addressed from a mathematical and strategic point of view.

Feel free to experiment with the two activities. You can use about an hour for each of the activities, depending on how much discussion time you decide to have. You can do just one of them or both, either one after the other or on different days.

As a general preparation, you only need to get familiar with the topic and the resources below, and prepare some interesting data to share with the students at specific moments.

Activity 1 - Melting the ice

This activity explores what would happen if the ice caps of the planet melted completely. At some point (maybe at the beginning and/or probably at the end) you should remind your students that this is a hypothetical scenario, posed to practice maths, physics, and Earth knowledge. Research indicates that during last century, the mean sea level rose about 16-21 cm, and it is expected to rise around 30 cm more during this century. While real data is concerning and must be addressed, here we will consider more extreme catastrophes.

1. **What comes first to your mind?**
   Start by asking the following question: "What do you think would happen to this city if the level of the ocean rises 2 meters?"
   Push the students to be creative with the answers. Would the coastline change? Would there be migration and refugees from the coastal cities? Lack of food production? Any economic effects? Would rivers change their course? Any effects on fauna? The goal is to make them think, not to give correct answers (but you can offer bits of information, for instance reference [4]).
   Change the value in the question. What if it rose 5 cm? What if it rose 10 m?

2. **If the North Pole melted**
   Ask what would happen if the ice in the North Pole melted. Leave time for them to come up with their own answers.
   After some time, remind them that the North Pole ice cap is an ice mass floating on the ocean (we will not consider Greenland as part of the ice cap). Propose the following thought experiment: Suppose that you have a glass of water filled to the rim, so it can't contain any more water. On the water there is an ice cube floating. Most of the ice is under the water but there is a corner of the ice cube sticking out of the water surface. Suppose then that you leave the glass with the water and the ice at room temperature until the ice melts. What will happen? Will the water spill over? Will the level decrease?

Question: What would happen to the sea level if the ice cap floating in the North Pole melted?
Answer: The sea level would remain exactly the same. In the example of the ice cube floating in the glass, the level would stay exactly the same and it would not spill. The explanation involves the physics of buoyancy (Archimedes principle). Imagine that you select a certain region of the water in the glass, a region located just below and in contact with the surface. Now imagine that the water inside of that region is transformed into ice. It would expand upwards, occupying part of the air above. But since the number of water molecules is the same, the mass of that ice is exactly the same as the water that was before, and therefore the upwards buoyancy force exerted by the rest of the water is the same as before and counteracts exactly the weight force of the ice. The ice would remain in buoyant equilibrium.

Therefore, the level of the oceans would remain absolutely the same if the North Pole ice cap melted. Of course, there would be many other significant consequences for the environment, the climate, and the planet as a whole.

3. **If the South Pole melted**

Now provide the fact that in the South Pole the ice cap is over the Antarctica landmass. Therefore, if the Antarctic ice cap melted, there would be a lot of water added to the oceans. Ask your students to calculate how much the sea level would rise. Some data is needed to give that estimation, you can leave your students to search for that data online, or you can give them the figures below. If you decide to give no data to your students, open a discussion about what data (and formulas) are needed to be found. In case that someone finds a final result online you can ask them to justify or reconstruct that result from more basic data.

Question: The ice in Antarctica covers a land area of 14 million km^2 and is on average 2 km thick. The Earth can be assumed to be a sphere of radius 6371 km and is filled up to 70% with water. Water is denser than ice, 1 m^3 ice is the equivalent of 0.9 m^3 water. If all the ice over the Antarctica landmass melted, how much would the sea level rise?

Answer: about 70 m

The volume of ice on the Antarctic landmass is

$$24 \text{ M km}^2 \times 2 \text{ km} = 28 \text{ M km}^3 \text{ ice.}$$

If that ice melted, it would convert into

$$28 \text{ M km}^3 \text{ ice} \times (0.9 \text{ m}^3 \text{ water} / 1 \text{ m}^3 \text{ ice}) = 25.2 \text{ M km}^3 \text{ water.}$$

From the other side, the surface of the Earth is

$$4 \times \text{pi} \times \text{radius}^2 = 4 \times 3.1416 \times (6371 \text{ km})^2 = 510 \text{ M km}^2.$$

If only 70% of that surface is water, that gives us a surface for the oceans of

$$510 \text{ M km}^2 \times 0.7 = 357 \text{ M km}^2.$$

Now, we can assume that all the melted ice would be accumulated on the same surface as the oceans cover. Since the volume equals the surface area times the height,

$$\text{Volume} = \text{Area} \times \text{height}$$

then the sea level height rise is just the quotient of the added volume of water divided by the surface of the oceans:

$$h = 25.2 \text{ M km}^3 / 357 \text{ M km}^2 = 0.0705 \text{ km} = 70 \text{ m.}$$

Thus, our estimation is that if all the ice in the Antarctica melted, the oceans level would rise by about 70 meters.

Now, could we improve the model? One option would be to take into account that not all the water would be piled up over the existing ocean, but some would be spread over some part of land of low altitude, flooding it.

Question: Assume that about 10% of the land would be flooded, and that the average altitude of that flooded region is 10 m. How much would the ocean rise in that case?
These two numbers are just a guess, change them to see if the result changes significantly.

Answer:
The surface of land is 30% of the Earth's surface.

$$510 \text{ M km}^2 \times 0.3 = 153 \text{ M km}^2$$

Now we have 10% of land flooded, that is,

$$153 \text{ M km}^2 \times 0.1 = 15.3 \text{ M km}^2$$

Since this flooded land has a mean altitude of 10 m, this means that the mean height of water above it will be h - 0.01 km. Here the shape of the inundated land is not important, as long as we know that the *mean* altitude is 10m.
Now, the volume of new water is calculated as:

$$\text{Volume} = \text{Area}_{oceans} \times h + \text{Area}_{flooded} \times (h - 0.01 \text{ km})$$

The volume of new water and the area of the oceans are the same as in the previous example. Solving for h gives

$$h = 68 \text{ m}$$

Thus, there is a very small difference. This is not surprising: a 10% of the land is only 3% of the surface of the Earth.
We can play with the two values as parameters. The extreme case is if the whole land had altitude zero, the rise would still be 49.4 meters, hence very high.

The reference [2] provides the answer of a sea level rise of 73.32 m, probably using more accurate data than our quick estimation. The final figure of that study is a sea level rise of 80.32m if we include Greenland and all other glaciers on Earth.

As we pointed out earlier, that is an unlikely catastrophic scenario. Predictions are for a 30 cm increase by the end of the century. Another factor that significantly affects the sea level is the thermal expansion of the water due to a warmer temperature. This not only raises the sea level but also accelerates the melting rhythm of the ice caps.

References:

You can use the following resources to enrich the discussion with real-world data and interactive visualizations:

1. Presentation with an overview of the current research on the ice caps melting.
   https://imaginary.org/program/simulating-the-melting-of-ice-caps
2. Real data from experts, presented for education context.
   https://serc.carleton.edu/eslabs/cryosphere/6b.html
3. Map app to observe which parts of the world would be covered by water after a sea level rise
   https://www.floodmap.net/
4. A total of 10 % of the world's population lives in low-lying coastal areas, i.e., less than 10 m above the current mean sea level.
   McGranahan, G., Balk, D., and Anderson, B.: The rising tide: Assessing the risks of climate change and human settlements in low elevation coastal zones, Environ. Urban., 19, 17–37, https://doi.org/10.1177/0956247807076960 , 2007.

Activity 2 - Orders of magnitude in questions about Earth

How can we find answers to big magnitude questions when we have little data? Sometimes statistics about a particular phenomenon are not available: maybe because the scope is too big (for instance the whole planet) or too small (your town, your school), and no institution has collected that data. Sometimes it is impossible to produce data because we cannot give an exact count (how many trees are there in the world?) or because it is hypothetical (how many trees would be needed to absorb the current CO2 emissions?) For those types of questions, we only look for a coarse estimation of the *order of magnitude* of the answer. Such estimates are often referred to as Fermi estimations.

There are many tricks that we can use to give answers to these questions. In this activity we explore this technique and apply it to some environmental questions related to planet Earth.

1. **Warm-up questions.**
   Ask your students to find an answer to the following questions:
   a. How much food is consumed per day around the world?
   b. How much garbage is produced per day around the world?
   c. How many liters of water are needed per day around the world just for individual consumption (excluding industries and agriculture)?

   For each one, they need to get an estimation from their own experience (how much food do I eat every day, how much garbage do I produce…), and you can provide them with statistics such as the world population (7 billion people). Ask them questions such as the variability of those figures. Is it the same consumption of food per person all across the world? What about the garbage? What about the garbage produced by industry, is it more or less than that of individuals? You can find some "official" answers to these questions by searching the Internet. Open a round of comments after giving the answers.

2. **Incomplete information questions.**
   Ask your students this new type of question where they'll need to obtain data from indirect sources and make simple models and estimations:
   a. How many liters of water does the school (or building you are in now) use each week?
   b. How many blades of grass are there on a football pitch (or any other sports field or garden)?
   c. How much CO_2 is exhaled by all students / participants per day?

3. **Deeper questions.**
   Bring up the following question:
   a. What do you think is the volume of all plastic bottles that end up in the ocean every day?

   Let your students guess (and help them with) the steps on decomposing the problem and making estimations. Some questions that may arise:
   - What is the correct order of magnitude? Kg? Tons? Millions of tons? Billions of tons?
   - How much plastic is produced by humanity?
   - How much of it ends in the ocean?

- You can assume (can you?) that plastic mismanaged, not recycled, is only from personal consumption, not industry, so you can try to estimate how much plastic a person consumes, and multiply by some population number.
- How many people live in coastal areas (countries with coasts)? You can assume that only countries with coast pollute the oceans (can you?).

For each step or piece in the decomposition you can try to make a guess or try to find it online.

Keep the debate open. The goal is to make them discuss the magnitudes and their relations. After some time, ask them to reach a consensus number.

Finally, use the article in reference [4] to give an expert answer: between 4.8 and 12.7 millions of tons of plastic debris thrown into the ocean in 2010, with the perspective of augmenting one order of magnitude (multiplying by 10) by 2025 if no measures are taken.

Open a final discussion round on the implications of the oceans pollution.

4. **Bonus project**
   Offer your students as a bonus project a final Fermi question that they can investigate by themselves (for example at home, with friends or family), and that cannot be found on the Internet. For instance, use local topics, such as: What is the CO2 balance of this city? That is, is it emitted to the atmosphere more or less CO2 than it is absorbed by the trees?

Resources

1. A cartoon explaining practically how to do a Fermi estimate
   https://what-if.xkcd.com/84/

2. Article that shows some tips about how to make good Fermi estimates:
   https://www.lesswrong.com/posts/PsEppdvgRisz5xAHG/fermi-estimates
   You can practice trying to solve these and other questions, and help your students with some hints.
   For instance, the article explains how to make an estimation of a magnitude from two bounds using the geometric mean, or the approximate geometric mean (AGM). This will add extra math content to the activity.

3. Examples of Fermi questions to use in the school (in general not related to Earth sciences):
   https://www.teachertoolkit.co.uk/2017/04/28/fermi-questions/

4. Research article that studies how much plastic is there in the ocean:
   Jambeck, J. R., Geyer, R., Wilcox, C., Siegler, T. R., Perryman, M., Andrady, A., Narayan, R., Law, K. L. (2015). *Plastic waste inputs from land into the ocean. Science, 347(6223), 768–771.* doi:10.1126/science.1260352

Create and Share!
Share your question, your thought process and discussions and your results using the hashtags **#idm314earth** and **#idm314**.

Further resources:

1. Climate Science educational resources (project TROP-ICSU): https://climatescienceteaching.org
2. Interactive climate simulator: https://en-roads.climateinteractive.org/scenario.html
3. Blog on Mathematics of Planet Earth: http://mpe.dimacs.rutgers.edu/blog/ (en)
4. More Fermi problems for the school: https://www.teachertoolkit.co.uk/2017/04/28/fermi-questions/
5. Resources on how to conduct question-based discussions in the school
    a. The right question at the right time
    b. Inquiry based maths education (from the EU project FIBONACCI)